

TRU Math Conversation Guide: A Tool for Teacher Learning and Growth

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A companion document, the *TRU Math Conversation Guide, Module A: Contextual Algebraic Tasks*, supports in-depth explorations of algebraic thinking, with a focus on complex modeling and applications problems. *Module A: Contextual Algebraic Tasks* is the first of a series of content-specific conversation guides aimed at supporting classroom engagement with centrally important mathematical ideas. The *TRU Math Conversation Guide Modules* will all be accessible at <http://ats.berkeley.edu/tools.html> and/or <http://map.mathshell.org/materials/index.php>.

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TRU Math Conversation Guide: A Tool for Teacher Learning and Growth¹

Teachers' work is remarkably complex—in particular when the goal is not just for students to master facts and procedures, but to **T**each for the **R**obust **U**nderstanding of **M**athematics. Because of this, there is *always* room for learning and growth, for *every* teacher regardless of prior training, years of experience, or current successes. Indeed, ongoing learning is the essence of teaching.

Our experience as teachers, coaches, and researchers has been that our most meaningful learning occurs when we interact with others, developing and sustaining relationships that push us to expand our vision of mathematics teaching, offer perspectives on our challenges that differ from our own, and respect our intelligence, skill, and intentions—as well as our need to continually grow. These supportive relationships not only help us to alter our practice but also to deepen our understanding of the complex work we are undertaking.

Unfortunately, much of the professional development that we have experienced has focused less on these aspects of learning and more on “experts” sharing best practices that we are supposed to simply import to our own classrooms. Some of this PD has been research-based, while some of it has not. In contrast, we are aiming for a professional development tool that builds on what teachers, coaches, and professional learning communities know.

What this tool is, and who it is for²

This conversation guide represents our best effort to use research to support teacher learning and growth in a way that accounts for both how people learn and the complexity of teaching practice. Instead of prescribing instructional techniques or tricks, we offer a set of questions organized around dimensions of teaching identified by research as critical for students' mathematics learning. The purpose of these questions is not to tell anyone how to teach, but to guide discussions between teachers and supportive others, fostering a process of *learning together*. We hope that the questions will support teachers to develop, articulate, and progress on their own learning agendas, through ongoing dialogue with coaches, administrators, colleagues, and others, in order to support the development of students' robust understanding of mathematics.

At the guide's core is a set of questions designed to facilitate discussions between teachers and supportive others (coaches, administrators, colleagues, etc.—as well as for teachers to reflect on their

¹ You are reading the first public version of this conversation guide. We hope that reflecting on teaching in the ways suggested here will be productive. We also welcome comments and suggestions for improvement. Please contact Nicole (NLL@berkeley.edu) and Evra (evra@berkeley.edu) with your feedback.

² This draft presents the domain-general part of the Conversation Guide, corresponding to Dimensions 1-5 of the TRU Math framework (see the *An Introduction to the TRU Math Dimensions* for more detail). The *TRU Math Conversation Guide Module A: Contextual Algebraic Tasks*, the first of a series of content-specific modules, focuses on focuses on contextual algebraic modeling tasks.

own between discussions with others). Importantly, the guide is designed for conversations that are grounded in classroom observations; it will work best when it is used by people who have some shared experience of an actual lesson, with actual students. The questions are organized along five dimensions:

- The Mathematics
- Cognitive Demand
- Access to Mathematical Content
- Agency, Authority, and Identity
- Uses of Assessment

These dimensions offer a way to organize some of the complexity of teaching so that we can focus our learning together in deliberate and useful ways. They include attention to mathematics content, mathematical practices, and students' developing identities as learners and doers of mathematics. There is necessarily some overlap between dimensions; rather than capturing completely distinct aspects of classroom life, each one is like a visual filter, placing different emphasis on the same phenomena. We encourage you to think about interactions between dimensions when it feels useful for you. The questions in the conversation guide will also direct your attention to particular kinds of overlap.

In the remainder of this document we provide an overview of each dimension; discussion questions for each dimension, for use by teachers and others to reflect on and plan instruction; and a set of suggestions for and examples of how to use the discussion questions.

We hope you will find the guide useful. Happy teaching and learning!

HOW TO USE THIS CONVERSATION GUIDE

Our field tests and our experiences as instructional coaches have led to a few suggestions that may help you make the most of this conversation guide. In this section, we share these suggestions and give some examples of how conversations using the guide might look.

1. Set a long-term learning agenda.

Complex learning—like learning how to teach for robust student understanding—has so many facets that it is easy to jump from one thing to another, without making clear progress on anything. Setting a long-term learning agenda can help us focus our energies, whether we're full-time classroom teachers or people who support classroom teachers. Opportunities to have deep conversations about practice are few and far between, but if we have a core learning agenda that we can return to again and again, we stand a better chance of leveraging all our strengths to learn together about something that matters.

You might decide to set a learning agenda together, or the teacher might already have something in mind. We strongly suggest that others not dictate what a teacher's learning agenda should be. Others might have suggestions based on what they know about a teacher's practice, but teachers should ultimately have ownership and control over their own learning agendas.

Some examples of long-term learning agendas might be, “This semester, I want to focus on getting students to share their reasoning, not just answers or steps students completed to get answers,” or “This year, I want to get better at engaging students who get frustrated and give up easily.” As you set your own learning agenda, it may be useful to read through the dimensions and discussion questions, to see if anything jumps out as particularly important or exciting.

2. Use the discussion questions like a menu. Pick and choose.

You might have noticed that there are a lot of questions! Our design assumes that you WILL NOT try to discuss every bullet, one by one, each time you use the guide. Instead, we hope you will identify areas of the guide that are appropriate for your learning agenda and return to these areas again and again. We expect that some of the questions will be difficult to answer – and that by discussing them together you will find new ways of understanding what is happening in your classroom, and come up with ideas for things to try to advance both teacher and student learning.

3. Have a conversation prior to each observation.

Schedules don’t always make this possible, but if you can, have a pre-observation conversation in which you clarify the goals not just for the lesson, but also for its observation component. Talk about goals for students, so that the observer has the same frame for the lesson as the teacher. But also remind each other of the *teacher’s* learning agenda, and discuss how the observer can be most helpful. What should she be looking for (e.g., recording the questions the teacher asks, or focusing on a particular student)? What kinds of interactions should she have with students? This “pre-brief” is one way of following through on the focus and organization that the learning agenda offers. Without it, it’s easy to get distracted when the actual observation happens. It’s also easy for the observer to notice things that are not interesting or important to the teacher, which are less likely to help the teacher learn and grow.

You’ll notice that the conversation guide includes prompts for “pre-observation” discussions, for example, “What opportunities will students have to make their own sense of important mathematical ideas?” (under Cognitive Demand). These prompts are designed to bring to the surface ideas about what is likely to happen in the lesson, given the tasks students will be given, the participation structures that will be used, and so on. This kind of anticipatory thinking might lead to tweaks in the lesson plan, but the primary intention is to establish common focus between the teacher and observer. This adds richness to the debrief after the lesson; everyone can then reflect on the ways that things worked out the way they were intended to, ways they were surprising, and next steps in light of that information.

4. Analyze teaching from the perspective of different “tenses.”

In addition to prompts for “pre-observation” discussions, this conversation guide includes prompts for “reflecting after a lesson” and “planning next steps.” For each dimension, there is a list of things to think about; each of these things can be considered from the perspective of each prompt. For example, the reflection prompt for the Access dimension reads, “Who did and didn’t participate in the mathematical work of the class, and how?” So after a lesson, you could think about “the range of ways that students [could and did] participate,” noticing that the main way the students participated during the whole-class part of the lesson was by raising their hands to respond orally to questions the teacher

posed, and that this actively involved about one third of the class. You could then extend this thinking to the planning prompt, “How can we create opportunities for each student to participate in the mathematical work of the class?”

Our idea is that planning should be firmly grounded in reflection on what has actually happened—and that it is worthwhile to make space for thinking about what actually happened, because this opens up possibilities for future action that might otherwise remain hidden. (More on this immediately below.) In addition, not everyone will have seen the same things, and sharing can enrich everyone’s understanding of what students were doing, thinking, and learning.

5. Ground discussion in specific details from the lesson or from student work.

We’ve all made statements like, “My kids seem to really get linear equations” or “They’re really struggling with fractions.” While these statements convey a picture of student understanding in a quick and concise way, they need to be followed up with more detailed information. Otherwise, it is difficult to make instruction responsive to student thinking, and easy to miss opportunities to build on students’ strengths or address their misconceptions. One way to make our observations more specific is to talk about content with as much detail as possible; for example, instead of saying “They’re really struggling with fractions,” you might observe that “Even though I’ve seen them do just fine with finding equivalent fractions and even adding them, they just seem to shut down every time they see a fraction,” or “they’re reducing fractions in a mechanical way, but they don’t seem to see that $\frac{4}{6}$ of a chocolate bar and $\frac{2}{3}$ of a chocolate bar represent the same amount.” Pressing for examples from the lesson or from previous lessons also makes observations more accurate and concrete, helping us get away from our general impressions and closer to actual student thinking. Talking about specific students—and ways that their thinking is or isn’t typical of the class—is another strategy. Not only does this strategy give us a more detailed and accurate picture of the thinking that is going on in our classrooms, but it also opens up instructional possibilities. For example, noticing that today, Jessica drew a really helpful picture to represent fractions could lead you to invite Jessica to share her method with the rest of class, creating a learning opportunity that is invisible in “They’re really struggling with fractions.” Finally, attending to particular students can help us think about patterns of marginalization in society at large (e.g., fewer resources for ELLs, or stereotypes that link race, gender, and mathematics ability), and how our classrooms might work to replicate or counter those patterns for our own students.

The Mathematics

Core Question: How do mathematical ideas from this unit/course develop in this lesson/lesson sequence?

Students often experience mathematics as a set of isolated facts, procedures and concepts, to be rehearsed, memorized, and applied. Our goal is to instead give students opportunities to experience mathematics as a coherent and meaningful discipline. This means identifying the important mathematical ideas behind facts and procedures, highlighting connections between skills and concepts, and relating concepts to each other—not just in a single lesson, but also across lessons and units. It also means engaging students with centrally important mathematics in an active way, so that they can make sense of concepts and ideas for themselves and develop robust networks of understanding.

The Mathematics

Pre-observation	Reflecting After a Lesson	Planning Next Steps
How will important mathematical ideas develop in this lesson and unit?	How did students actually engage with important mathematical ideas in this lesson?	How can we connect the mathematical ideas that surfaced in this lesson to future lessons?

Think about:

- The mathematical goals for the lesson.
- What connections exist among important ideas in this lesson and important ideas in past and future lessons.
- How math procedures in the lesson are justified and connected with important ideas.
- How we see/hear students engage with mathematical ideas during class.
- Which students get to engage deeply with important mathematical ideas.
- How future instruction could create opportunities for more students to engage more deeply with mathematical ideas.

Cognitive Demand

Core Question: What opportunities do students have to make their own sense of mathematical ideas?

We want students to engage authentically with important mathematical ideas, not simply receive knowledge. This requires students to engage in productive struggle. They need to be supported in these struggles so that they aren't lost, but at the same time, support should maintain students' opportunities to grapple with important ideas and difficult problems. Finding a balance is difficult, but our goal is to help students understand the challenges they confront, while leaving them room to make their own sense of those challenges.

Cognitive Demand

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities will students have to make their own sense of important mathematical ideas?	What opportunities did students have to make their own sense of important mathematical ideas?	How can we create more opportunities for students to make their own sense of important mathematical ideas?

Think about:

- What opportunities exist for students to struggle with mathematical ideas.
- How students' struggles may support their engagement with mathematical ideas.
- How the teacher responds to students' struggles and how these responses support students to engage without removing struggles.
- What resources (other students, the teacher, notes, texts, technology, manipulatives, various representations, etc.) are available for students to use when they encounter struggles.
- What resources students actually use and how they might be supported to make better use of resources.
- Which students get to engage deeply with important mathematical ideas.
- How future instruction could create opportunities for more students to engage more deeply with mathematical ideas.
- What community norms seem to be evolving around the value of struggle and mistakes.

Access to Mathematical Content

Core Question: Who does and does not participate in the mathematical work of the class, and how?

All students should have access to opportunities to develop their own understandings of rich mathematics, and to build productive mathematical identities. For any number of reasons, it can be extremely difficult to provide this access to everyone, but that doesn't make it any less important! We want to challenge ourselves to recognize who has access and when. There may be mathematically rich discussions or other mathematically productive activities in the classroom—but who gets to participate in them? Who might benefit from different ways of organizing classroom activity?

Access to Mathematical Content

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities exist for each student to participate in the mathematical work of the class?	Who did and didn't participate in the mathematical work of the class, and how?	How can we create opportunities for each student to participate in the mathematical work of the class?

Think about:

- What range of ways students can and do participate in the mathematical work of the class (talking, writing, leaning in, listening hard; manipulating symbols, making diagrams, interpreting graphs, using manipulatives, connecting different strategies, etc.).
- Which students participate in which ways.
- Which students are most active when, and how we can create opportunities for more students to participate more actively.
- What opportunities various students have to make meaningful mathematical contributions.
- Language demands and the development of students' academic language.
- How norms (or interactions, or lesson structures, or task structure, or particular representations, etc.) facilitate or inhibit participation for particular students.
- What teacher moves might expand students' access to meaningful participation (such as modeling ways to participate, providing opportunities for practice, holding students accountable, pointing out students' successful participation).
- How to support particular students we are concerned about (in relation to learning, issues of safety, participation, etc.).

Agency, Authority, and Identity

Core Question: What opportunities do students have to explain their own and respond to each other’s mathematical ideas?

Many students have negative beliefs about themselves and mathematics, for example, that they are “bad at math,” or that math is just a bunch of facts and formulas that they’re supposed to memorize. Our goal is to support all students—especially those who have not been successful with mathematics in the past—to develop a sense of mathematical agency and authority. We want students to come to see themselves as mathematically capable and competent—not by giving them easy successes, but by engaging them as sense-makers, problem solvers, and creators of mathematical ideas.

Agency, Authority, and Identity

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities exist in the lesson for students to explain their own and respond to each other’s mathematical ideas?	What opportunities did students have to explain their own and respond to each other’s mathematical ideas?	What opportunities can we create in future lessons for more students to explain their own and respond to each other’s mathematical ideas?

Think about:

- Who generates the mathematical ideas that get discussed.
- Who evaluates and/or responds to others' ideas.
- How deeply students get to explain their ideas.
- How the teacher responds to student ideas (evaluating, questioning, probing, soliciting responses from other students, etc.).
- How norms around students' and teachers' roles in generating mathematical ideas are developing.
- How norms around what counts as mathematics (justifying, experimenting, practicing, etc.) are developing.
- Which students get to explain their own and respond to others' ideas in a meaningful way.

Uses of Assessment

Core Question: What do we know about each student's current mathematical thinking, and how can we build on it?

We want instruction to be responsive to students' actual thinking, not just our hopes or assumptions about what they do and don't understand. It isn't always easy to know what students are thinking, much less to use this information to shape classroom activities—but we can craft tasks and ask purposeful questions that give us insights into the strategies students are using, the depth of their conceptual understanding, and so on. Our goal is to then use those insights to guide our instruction, not just to fix mistakes but to integrate students' understandings, partial though they may be, and build on them.

Uses of Assessment

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What do we know about each student's current mathematical thinking, and how does this lesson build on it?	What did we learn in this lesson about each student's mathematical thinking? How was this thinking built on?	Based on what we learned about each student's mathematical thinking, how can we (1) learn more about it and (2) build on it?

Think about:

- What opportunities exist for students to develop their own strategies and approaches.
- What opportunities exist for students to share their mathematical ideas and reasoning, and to connect their ideas to others'.
- What different ways students get to share their mathematical ideas and reasoning (writing on paper, speaking, writing on the board, creating diagrams, demonstrating with manipulatives, etc.).
- Who students get to share their ideas with (e.g., a partner, the whole class, the teacher).
- How students are likely to make sense of the mathematics in the lesson and what responses might build on that thinking.
- What things we can try (e.g., tasks, lesson structures, questioning prompts such as those in FALs) to surface student thinking, especially the thinking of students whose mathematical ideas we don't know much about yet.
- What we know and don't know about how each student is making sense of the mathematics we are focusing on.
- What opportunities exist to build on students' mathematical thinking, and how teachers and/or other students take up these opportunities.

A NOTE: What We Mean By “Important Mathematical Ideas”

“Important mathematical ideas” are notoriously hard to define. Which ideas are important? Which are not? What even counts as an “idea”? Who should have the authority to decide? Our intention with the conversation guide is to support discussions about these questions rather than to offer answers. To us, it is much more important to work together to push our students and ourselves as educators toward more interconnected and fundamental understandings of mathematics than to decide exactly which ideas are most important. This pushing is crucial, though, because traditional views of school mathematics—and many of today’s textbooks and standards documents—define mathematics in terms of isolated topics, skills, and sub-skills. Thinking about the progression of mathematical ideas as “Day 1: Add and Subtract Fractions With Like Denominators; Day 2: Multiply Fractions; Day 3: Divide Fractions; Day 4: Add and Subtract Fractions with Unlike Denominators” (a typical textbook progression) makes it difficult to develop conceptual understanding and a sense of meaning behind all of the mechanics. This is both untrue to mathematics as a discipline and alienating for many students.

When we reflect on and plan instruction, we find the questions below useful. They help us shift our focus from topics and skills to ideas that feel mathematically important. We hope they will be helpful for you as well.

- What do we want students to understand about the relevant mathematical objects (fractions, negative numbers, the coordinate plane, triangles, etc.) in this lesson? In this unit?
- What mathematical relationships, patterns, or principles do we want students to understand in this lesson? In this unit?
- How might students connect math ideas in this lesson/unit with ideas that came before or will come later? Are there overarching principles or relationships or patterns that they might work toward understanding?
- What are different ways of representing the math in this lesson/unit? How might different representations be connected to each other and how might these connections deepen our students’ understanding?
- How do the ideas we’re considering develop across multiple lessons/units?
- What are some ways to make connections to this idea in different lessons/units/content areas?

Some examples of math ideas that might be considered “important”:

- Area and perimeter are fundamentally different measurable attributes of two-dimensional shapes. It is possible to change shapes such that neither, one, or both of these attributes change. For some families of shapes, there are interesting relationships between them.
- Relationships between two variables can be represented using equations, tables, graphs, and verbal descriptions. Parameters of the relationship between the variables (e.g., the rate of change) can be identified in each of these representations and connected across representations.
- Right triangles have special properties that are different from the properties of other triangles. These properties give us special access to information about things like angle measures and side lengths in particular right triangles.
- Many sets of changing quantities are proportionally related. This means that certain aspects of the relationship are constant and unchanging, which allows us to use the relationship to determine one quantity given the other.

One characteristic of all of these ideas is that they go beyond naming topics and skills. For example, we might know that we want to “cover proportional relationships” in a particular unit, or that we want students to be able

to solve proportions. However, without consideration of what important mathematical ideas *about* proportional relationships we want our students to make sense of, we are likely to miss opportunities to support students to build conceptual understandings, to make connections, and to develop a sense of ownership over mathematical ideas.

Our hope is that as teachers and observers think together about teaching, they can continuously push each other to think about the mathematics that students need to learn in bigger, deeper, richer and more interconnected ways. So while our discussion questions frequently refer to “important mathematical ideas” as though there were a set list of such ideas somewhere that you could simply consult, we hope that you will instead find ways to explore and interrogate what “important mathematical ideas” means to you.